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SOME STATISTICAL TESTS IN THE STUDY OF TERRAIN MODELING

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Abstract

We decided to create statistical models of the course elevation profiles of the U.S.Army's vehicle test courses at the Aberdeen Proving Grounds, Aberdeen, Maryland, and at the Yuma Test Facility, Yuma Arizona.
As a first step in creating the models, we decided first to test the data, the output from various profilometer runs, to see if the course elevations met the assumptions made in the past and in current models. The assumptions are that the course elevation profiles, considered as time series (signals), are Linear, Stationary and Gaussian. Because of past analyses we had suspected that these assumptions were not met.
In the body of the text we attempt to explain the statistical terminology and how the statistics are used. The appendices are reserved for the technicalities of the tests.
To test for linearity a test due to Keenan was chosen. This procedure was chosen because it tests linearity against bilinearity, and because it is easy to implement. The gist of Keenan's test is to assume a model that contains both linear and bilinear terms and to see if the square of the output is uncorrelated with the residuals of the model minus the data . The well known test that checks whether one distribution agrees with another, the Kolmogorov Smirnoff test, was used to check for Gaussinity. Stationarity was tested by breaking up the data into segments of equal length and checking whether the variance was the same for each segment.
The data for each of the test courses was broken up into segments of equal length and the statistical tests run on each segment. The results are summarized in two tables whose contents will be explained in the text.
The results show that Belgian Block course is linear and Gaussian but not stationary, and the course, Perryman3, however, is neither linear, nor Gaussian, nor stationary.
The outcome of this portion led us to try to model Belgian Block as a course that was almost stationary. Since the signal is not stationary, it does not have a spectral density, but it has a time(distance)dependent spectral density that changes along with the data and it is called a process with an" Evolutionary Spectrum". A very well behaved such process, an " Uniformly Modulated " process, was used to model Belgian Block. This analysis is completed and a report on it will appear elsewhere.

§1. INTRODUCTION.

The purpose of the work described herein was to create models of profiles of the courses on which the United States Army tests its' vehicles. These models were to be simple, transparent, usable, and most important of all statistically based, oriented, and valid.

Since applied statistics is performed on real data obtained from real measurements, tests, or experiments, in §2 we include detailed description of the background of the data collection. Where and why the Army collects data are reported and the Army's extended collection effort is described. §2 also contains the manner in which the data is taken and how the data is processed before any analysis is begun. Informal definitions and heuristics of the statistical tests are given at the end of this section.

In §3 we explain the methods of the statistical tests which we applied to the data for the purpose of modeling. In §4 the results of our statistical tests are summarized.

For flow of the narrative and ease of its reading, technical details of the statistical methods are left in the appendix at the end.

§2 DATA COLLECTION AND HEURISTICS.

The United States Army has been a pioneer in developing the standards by which terrain topography is characterized; it has been the leader in building computer models that incorporate topographic properties. The Army has been at the vanguard in constructing test courses that emulate the real world topography on which military vehicles travel. At the Army's Aberdeen Test Center (ATC) and Yuma Proving Ground (YPG), miles of test courses with various attributes are available for determining vehicle durability and mobility characteristics in temperate and desert environments. Each course is designed to challenge a particular military vehicle capability. Examples are hilly cross country for power train issues, level courses with incremental differences in surface roughness for long term durability issues, specialty vibration courses for developing vibration response envelopes, and specialty obstacles such as bridging devices, ditch profiles, "frame twisters", and soil

Figure 1: US Army Aberdeen Test Center Munson Test Area

bins to access vehicle mobility. An aerial photo of Act's Munson Test Area is shown in Figure 1.

(A) PROFILE-MEASURING EQUIPMENT

The course profiles are measured with a profilometer which is described in Army document([9]). The part of that document which describes the working of the profilometer is quoted, " . . . . a profilometer system which consists of a wagon type trailer (4 wheels with 2 axles) which is drawn by a tow vehicle (Fig 2), a data-acquisition subsystem, and mobile PC-computer-based data analysis system. The profilometer front axle is free to rotate about the yaw axis, but is constrained from other motion relative to the frame. A linkage to the draw bar is employed to constrain the front axle and align the wheels parallel to the drawbar. The rear axle is free to rotate about the roll axis and is constrained from all other motion relative to the frame. This system provides a platform which is articulate enough to conform to the anticipated terrain and to follow the tow vehicle. There are no compliant suspension components between the axles and the frame."

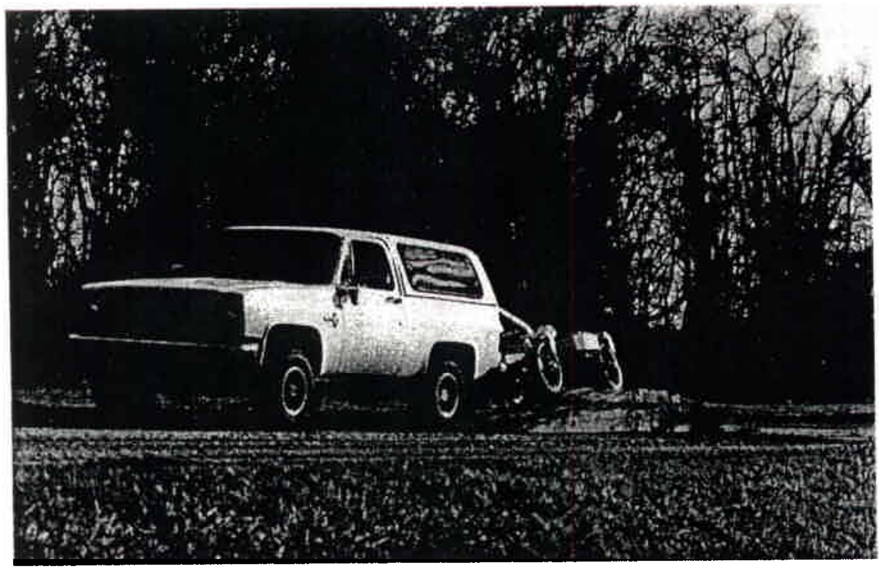

Figure 2: ATC Profilometer

"The profilometer contains an inertial gyroscope to measure pitch and roll angle and an ultrasonic distance measuring device to measure the vertical distance between the frame and the terrain. The vertical gyro also provides a signal to a stabilized platform so that the ultrasonic subsystem always points along the vertical. A shaft encoder provides a pulse output every 0.1 inches of travel. A counter accumulates the pulses and an interrupt is issued to activate the data acquisition system at a programmable distance (currently 3 inches)."

(B) DATA PROCESSING.

Before the model building process can begin the measurements obtained from the profilometer are converted to the data used in the model building. Simply the raw measurements are processed.

The profile of a road surface is calculated from the pitch and roll angles and the ultrasonic subsystem outputs. The pitch angle data are first used to

determine the locus of the profilometer chassis midpoint's motion. A set of data are acquired at regular intervals of travel along the surface, from which the locus of travel can be calculated .The calculations giving the locus of travel give results that are not at equally spaced intervals required by the analyses that follow. Linear interpolation is used to obtained the required equally spaced points. The dynamics caused by the profilometer which is not a perfect transfer function are removed by digital filtering .The low spatial frequencies added by the calculation are removed by detrending .It is generally accepted that wavelengths greater than sixty feet have little effect on vehicle dynamics so these are removed by filtering.

The data on which the model building begins is now ready.

(C) STATISTICAL TESTS.

A program of statistically characterizing the Army's test courses at Yuma and ATC was undertaken.The height elevation profiles of each course, measured at fixed distances, were considered as sample paths of a time series. A time series is a collection of observations, in this case course elevations, taken at regular intervals([8]). Here the intervals are distances from a fixed point.

The objectives of the time series modeling and analysis were: to create statistical models of each test course, to ascertain and quantify the roughness of each course, and to analyze how each course effects the dynamics of vehicles traversing that course. Further each such model was to be as simple as could be while accurately describing the crucial and salient features of the course. In addition, the numerical results of each model were to be used as input to computer based vehicle dynamic models or as input to mechanical devices such as various shakers that simulate the dynamics of a vehicle.

In order to create such statistical models of the course elevations, a series of statistical operations were taken. The first was to run a series of statistical tests that check whether the standard and current assumptions that are made about the time series (the course elevations) were reasonable. The usual assumptions made about a time series are that the series is linear, stationary,and Gaussian . Heuristically a statistical test is an assessment of some claim made about the statistical structure given the evidence that is

the data. For example that the mean of a distribution or a population is zero can be tested by well known procedures.([1]) The test statistic for the mean is usually the average of the values of a sample. For this data, what is to be tested first is whether the road elevation profile could be considered a sample from a linear time series. Heuristically a time series is linear if it can be represented as a linear combination of past values of a random series. A time series is stationary if the first two moments that is the mean, the variance, and the correlation are constant (invariant with) over time. A test to ascertain whether the profiles are stationary will be run. A process (represented by a time series) is Gaussian if any finite set of variables (finite number of members of the time series) has a multivariate normal or Gaussian distribution. Gaussinity will also be tested on the time series profiles.

There are many reasons for running these tests. For example many analyses of a signal or time series use the power spectral density (PSD) of the signal. The assumption of the existence of a PSD presupposes stationarity([3]). The elevation profiles being stationary also implies that if roughness is measured by variance then the course is uniformly rough along its length. Linearity is a very useful property. It simplifies the representation of the series and makes it understandable and computable. Normality allows classical statistical inference to be used to run tests and check hypotheses. Obtaining intuition about the data, its structure and peculiarities, is important in the model building and some of this can be done by the formal testing process([2]), ([4]).

A description of each of the statistical tests that were used and the results of each test will be given.

In order to check the statistical and numerical methods as well as the computer programs that implemented them, data from two of the Army's courses' elevations were used. Belgian Block is a man made course constructed of bricks and is relatively benign. Perryman3 is a course that emulates the characteristics of traveling off road and therefore is very rough. Up close photographs of each is given in Figures 3 and 4 and 5.

§3 STATISTICAL TESTS FOR STATIONARITY, GAUSSIANITY, AND LINEARITY.

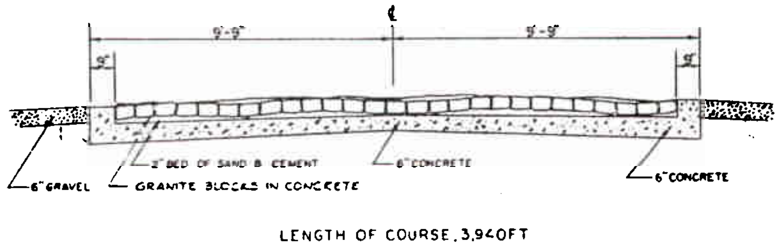

Figure 3: A Cross Sectional Representation of Belgian Block

Figure 4: A Snake's View of Belgian Block

Figure 5: Perryman3 Cross Country Course

Some general statistical properties of a given set of data need to be discovered before time series models can be successfully built and it can be said that these models represent the data or reflect its inherent structure. To discover some of these properties statistical tests are applied. Using some well known statistical tests it can be determined whether the data behaves like a Stationary Process, a Linear Process or a Gaussian Process.

(A) TEST FOR LINEARITY.

A procedure due to Keenan([7]) is used to test for the linearity of a time series. What follows is a brief description of Keenan's test and how it will be used. Suppose $X_1, X_2, \cdots, Xn$ is a stationary time series. It will be assumed that it is bilinear which means that it can be expressed as an expansion consisting of two parts. In symbols,

$$X_t = \mu + \sum_{i=-\infty}^{\infty} b_i e_{t-i} + \sum_{i,j=-\infty}^{\infty} b_{ij} e_{t-i} e_{t-j}. (1)$$

The ϵ_t is a random series (a series of random shocks), $\{b_i\}$ and $\{b_{ij}\}$ is each a sequence of constants and μ is a constant. In case X_t has a representation as a linear model, the constants b_{ij} in equation(1) would be zero and consequently the data can be approximated by an autoregressive model of a very large order M, written $AR(M)$.([8]) That is

$$X_t = a_0 + \sum_{j=1}^{M} a_j X_{j-t} + \epsilon_t, (2)$$

where $\{a_j\}$ are constants and M is as described.

Keenan's test which is derived from a method due to Tukey has the following as its principal idea. Perform a linear regression with X_t as the dependent variable (response) and $\{1, X_t - 1, \cdots, X_{t-M}\}$ as the dependent variables (predictors) and let $\{\epsilon_t\}$ be the residuals of the regression([6]). If the model is linear, then the residuals of the regression will be independent of X_t^2.

Another way is to say that the non-linearity is contained in the residuals of the regression. That is if it is bilinear by assumption then the $\{\epsilon_t\}$ in the regression will be statistically correlated with X_t^2. This as usual in regression analysis yields a test statistic that has an F distribution ([6]). If the value of this measure, the test statistic, is too large under the hypothesis of linearity, then we accept that a bilinear model better suits the data. More details are in Appendix(A) .

(B) TEST FOR GAUSSIANIT.

It is hypothesized that the data have a Gaussian distribution. A standard test for the Gaussinity (Normality) of a distribution, when the mean μ and the variance σ^2 are known is Kolmogorov's goodness of fit test. A full description is given in Appendix(B). The empirical distribution function, $EDF(x)$, of the data sample, $\{x_1, x_2, \cdots, x_n\}$, is a real valued function of x. The function $EDF(x)$ equals the proportion (fraction) of the x_i's in the sample that are less than or equal to x. So the $EDF(x)$ looks like a ladder with steps of height $k(x)/n$, $k(x)$ are the number of x_i's smaller than or equal to x and of course n is size of the sample. Kolmogorov's test statistic measures the distance between the graphs of the given hypothesized distribution $F(x)$ and the empirical distribution function of the data $EDF(x)$. Here the distance between the graphs will be denoted by D_n and the test statistic is given in the formula

$$D_n = \sup_x |EDF(\mu + \sigma x) - \Phi(x)|,$$

where $EDF(x)$ is the empirical distribution function from data $x_1, x_2, \cdots, x_n$ and $\Phi(x)$ is the standard Gaussian distribution.

Note sup means the greatest for all values of x. It can be shown that as the sample size n gets larger the step ladder gets closer to the distribution. That means D_n gets very small so that a large value of D_n lends little credence to the hypothesis. In this case that the distribution is Gaussian([1]). Further details are in Appendix(B).

(C) TEST FOR STATIONARITY.

The stationarity of a time series implies in particular that there is no

change in its variance. This property will be used to test for the stationarity of the data set describing the road profile. Of course most terrain profiles,in particular the long ones, are easily seen not to be stationary. However many profiles of shorter terrains do not have constant variance either and are therefore not stationary. To see this, the data of the terrains was divided into computably convenient segments of 1000 points (about 250 feet).The sample variance of the elevation profiles of each segment was computed. A significant change in variance was observed from segment to segment for each of the two profiles.

§4 RESULTS OF THE TESTS.

The three tests just described were applied to the profilometer data taken of Belgian Block and Perryman3. For convenience of computation the data measured on each course was divided into segments of 1000 points (about 250 feet in length).Since the data is taken monthly, an arbitrary month's data was chosen. For Belgian Block, it was February 2000. For Perryman3, it was March 2000. For both, the data was obtained from the left wheel. Tables 1 and 2 are a summary of the results obtained for the three statistical tests. Those for Belgian Block are in Table 1. The columns are explained by the column labels. The fourth column has heading $\hat{F}$ which refers to the value of the F test in the test for linearity.The 95confidence limit is 3.84. The column labeled D_n^* refers to the test for normality and the critical value is 0.028. The change in the variance can be seen in the relative magnitudes of the values in the last column. For Belgian Block, it can therefore be concluded that because only one segment of the eleven exceeds the critical value that it may safely be assumed that Belgian Block can be modeled as a linear process. For Gaussinity, only segments numbered 2,8,10,11,exceeded the critical value slightly. It seems reasonable to assume that the data has a Gaussian distribution. The obvious non constancy of the variance for the data obtained from Belgian Block leads to the conclusion that it is not Stationary. The three statistical tests applied to the profilometer data taken of Perryman3 is reported in Table 2 .The conclusion is that it is a very rough and unpredictable course and is neither linear nor Gaussian, nor stationary.

Table 1
Results of Tests for Linearity, Gaussianity and Variance for Belgian Block bb2l

5% Critical values for $\hat{F}$ is 3.84 and for D_n^* is 0.028

Segment	From	To	$\hat{F}$	D_n^*	Varience
1	1	1000	0.466	0.0301	0.0061
2	1001	2000	0.0373	0.0488	0.0058
3	2001	3000	0.206	0.0158	0.0041
4	3001	4000	1.5528	0.0245	0.0039
5	4001	5000	0.997	0.0181	0.005
6	5001	6000	1.1612	0.0182	0.0039
7	6001	7000	0.3775	0.0258	0.0031
8	7001	8000	6.4154	0.0444	0.0039
9	8001	9000	0.6837	0.0202	0.0026
10	9001	10000	0.2713	0.0276	0.0037
11	10001	11000	0.5131	0.0342	0.0056

Table 2
Results of Tests for Linearity, Gaussianity and Variance for Perryman cc3-3l

5% Critical values for $\hat{F}$ is 3.84 and for D_n^* is 0.028

Segment	From	To	$\hat{F}$	D_n^*	Varience
1	1	1000	4.0648	0.1111	0.0065
2	1001	2000	4.6421	0.1646	0.0567
3	2001	3000	8.0501	0.1328	0.0463
4	3001	4000	6.6891	0.1125	0.0476
5	4001	5000	1.8847	0.2077	0.0892
6	5001	6000	3.9111	0.1158	0.0643
7	6001	7000	3.2387	0.1226	0.0567
8	7001	8000	1.8346	0.1207	0.0596
9	8001	9000	4.7211	0.1446	0.0826
10	9001	10000	6.8518	0.1765	0.047

(**Table 2** continued)

11	10001	11000	4.6364	0.1283	0.0701
12	11001	12000	3.0133	0.0884	0.0512
13	12001	13000	2.0033	0.1054	0.0599
14	13001	14000	0.9298	0.168	0.0924
15	14001	15000	3.2387	0.1074	0.0525
16	15001	16000	1.2962	0.1034	0.0566
17	16001	17000	2.694	0.101	0.117
18	17001	18000	1.9163	0.1322	0.0644
19	18001	19000	1.1984	0.158	0.0545
20	19001	20000	4.1102	0.1481	0.0809
21	20001	21000	2.1901	0.1354	0.0653
22	21001	22000	1.5255	0.1578	0.0871
23	22001	23000	0.7957	0.1336	0.0623
24	23001	24000	0.3339	0.1417	0.0789
25	24001	25000	1.2467	0.0947	0.044
26	25001	26000	8.2848	0.1365	0.0521
27	26001	27000	3.2814	0.1805	0.0797
28	27001	28000	0.7493	0.1	0.0107
29	28001	29000	15.0203	0.1415	0.0231
30	29001	30000	3.0142	0.1988	0.0624
31	30001	31000	2.7729	0.1436	0.0815
32	31001	32000	1.0277	0.1423	0.0646
33	32001	33000	2.022	0.1547	0.0666
34	33001	34000	0.4491	0.1982	0.0913
35	34001	35000	2.9958	0.0691	0.0515
36	35001	36000	0.7654	0.119	0.0912
37	36001	37000	0.1792	0.1045	0.0998
38	37001	38000	0.0975	0.095	0.0796
39	38001	39000	1.1017	0.1418	0.0983
40	39001	40000	0.4403	0.1035	0.0772
41	40001	41000	1.5212	0.0967	0.0727
42	41001	42000	0.3479	0.0783	0.108
43	42001	43000	0.1929	0.0569	0.0788
44	43001	44000	3.9769	0.04727	0.0903

(**Table 2** continued)

45	44001	45000	1.7594	0.1067	0.0931
46	45001	46000	1.1142	0.1308	0.0663
47	46001	47000	1.5448	0.115	0.1005
48	47001	48000	1.1961	0.0631	0.0596
49	48001	49000	0.6876	0.099	0.0577
50	49001	50000	0.5896	0.0893	0.0365
51	50001	51000	0.025	0.0795	0.072
52	51001	52000	1.0038	0.0781	0.0818
53	52001	53000	0.2927	0.1215	0.0874
54	53001	54000	1.8961	0.0628	0.0321
55	54001	55000	3.5116	0.138	0.0438

§5 CONCLUSION.

The next step in the process is to build a model that incorporates the knowledge obtained from the diagnostic testing just described. It was learned that a model not too far from stationary with a variance that changes moderately over time can be used to describe the course, Belgian Block. Such a model is Priestley's uniformly modulated model. The uniformly modulated process has a time dependent spectral density function that is a product of a fixed density function times a function of time alone .The function of time represents the changes in variance of the profile. How this modeling was done will be reported elsewhere. Since there are many varied test courses at ATC and Yuma, it might be necessary to characterize them using other statistical models. This paper describes the statistics of one step in the building of models that characterize terrain profiles. Much has been learned about the needed models and much more modeling needs to be done.

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APPENDIX.

(A) KEENAN'S TEST FOR LINEARITY.

Here we describe Keenan's procedure for testing the null hypothesis of a linear model

$$X_t = \mu + \sum_{i=-\infty}^{\infty} b_i e_{t-i}$$

against the alternative hypothesis of a bilinear model

$$X_t = \mu + \sum_{i=-\infty}^{\infty} b_i e_{t-i} + \sum_{i,j=-\infty}^{\infty} b_{ij} e_{t-i} e_{t-j}.$$

where $\{\epsilon_i\}$ are independent identically distributed random variables with zero mean and variance σ^2. We assume that, under the null hypothesis, X_t can be well approximated by an $AR(M)$ model for some large M.

$$X_t = a_0 + \sum_{j=1}^{M} a_j X_{j-t} + \epsilon_t,$$

where $\{a_j\}$ are constants.

Step 1: Regress X_t on $\{1, X_{t-1}, \cdots, X_{t-}M\}$. M is a large but fixed integer. Calculate the predicted values called $\{\hat{X}_t\}$ and let the estimated residuals be called $\{\hat{\epsilon}_t\}$, for $t = M+1, M+2, \cdots, n$. Calculate the residual sum of squares $\sum_t(\hat{\epsilon}_t^2)$.

Step 2: To remove the affects of the correlation of $\{1, X_{t-1}, \cdots, X_{t-M}\}$ on $\hat{X}_t^2$, we regress $\hat{X}_t^2$ on $\{1, X_{t-1}, \cdots, X_{t-M}\}$ and calculate the resulting residuals $\{\hat{\xi}_t$ for $t = M, M+1, \cdots, n$.

Step 3: Regress $\hat{\epsilon}_t$ on $\hat{\xi}_t$ and calculate the regression coefficient . $\hat{\eta}_0$. Set $\hat{\eta} = \hat{\eta}_0(\sum_{t=M+1}^{n} \hat{\xi}_t^2)^{1/2}$. Calculate

$$\hat{F}_{1,n-2M-2} = \frac{\hat{\eta}^2(n-2M-2)}{\sum(\hat{\epsilon}_t^2) - \hat{\eta}^2}.$$

$\hat{F}_{1,n-2M-2}$ has an F-distribution with $(1, n-2M-2)$ degrees of freedom and has approximately a χ^2 distribution when n is large. For convenience $\hat{F}_{1,n-2M-2}$ will be denoted by $\hat{F}$.

Step 4: Reject the null hypothesis for large values of $\hat{F}$.

(B) TEST FOR GAUSSIANITY.

Assume the distribution is normal with mean μ and variance σ^2 known. Let

$$D_n = \sup_x |EDF(\mu + \sigma x) - \Phi(x)|.$$

As before, EDF is the empirical distribution function, Φ is the Gaussian distribution function, and D_n is known as the Kolmogorov statistic.

However the real value of μ and σ^2 are unknown and so each is replaced by its common and well known sample estimate and these estimates are known as $\bar{x}$ and $\hat{\sigma}^2$ respectively and the following is computed instead.

$$D_n^* = \sup_x |EDF(\bar{x} + \hat{\sigma}x) - \Phi(x)|$$

If the value of D_n^* is large, then the data does not support the contention that the distribution is Gaussian. In this case, the critical values for n= 1000 are given in Table (). As before, for a confidence of 95% set $\alpha = (1 - .95)$, k_α^* are the critical values.

$$\alpha = 0.05, \quad k_\alpha^* = 0.028$$
$$\alpha = 0.01, \quad k_\alpha^* = 0.033$$

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